C207; Project Set #4

due 3/21/2013

A Compton Monte Carlo

An intuitive and commonly applied method for modeling inverse Compton emission is to us a Monte Carlo approach, in which the radiation field is represented by discrete, individual packets of photons. The fate of each photon packet is determined by "rolling the dice", i.e., choosing random numbers drawn from the appropriate probability distributions. Here you will write your own simple Monte Carlo code, which could in principle be used to model real data. This is a numerical problem, so please provide a printout of your code, in addition to your plots.

As a concrete example, we can consider the interacting supernova discussed in PS 3. The supernova produces optical photons, which then travel through a shell of shocked circumstellar gas, where they may be scattered by non-thermal, relativistic electrons. Assume that the supernova emits a luminosity of $L_{\rm s} = 10^{43}$ egs s^{-1}. The luminosity carried by each Monte Carlo packet in our code is then $L_{\rm p} = L_{\rm s}/N_p$, where N_p is the number of photon packets we choose to use. We will assume for simplicity that all photons are emitted with the same frequency $h\nu_{\rm in} = \epsilon_{\rm in} = 1$ eV. These photons travel through a shocked shell of relativistic electrons with optical depth $\tau = 0.01$.

The Monte Carlo method proceeds by generating and scattering photon packets one by one. The energy of a scattered photon is given by applying two Doppler shifts – one into the rest frame, and one back out – which gives a final outgoing energy

$$\epsilon_{\rm out} = \epsilon_{\rm in}\gamma^2(1-\beta\mu_{\rm in})(1+\beta\mu'_{\rm out}) \tag{1}$$

where $\mu_{\rm in}$ is the incoming angle (in the lab frame) and $\mu'_{\rm out}$ is the outgoing angle (in the rest frame). We simulate a scattering by sampling the incoming and outgoing directions randomly[1]. Assuming for simplicity that both are isotropic, the distribution is uniform in $\mu = \cos\theta$, and so we can sample each angle as

$$\mu = 1 - 2R_i \tag{2}$$

where R_i is a random number[2] distributed uniformly between $(0,1)$. This expression gives the right range for μ of -1 to 1. After the random scattering, the packet has a new luminosity

$$L_{\rm out} = L_p \frac{\epsilon_{\rm out}}{\epsilon_{\rm in}}(1-e^{-\tau}) \tag{3}$$

[1] Since we are considering $\tau \ll 1$, we will only need to scatter each packet once.

[2] I give some of the syntax for generating random numbers and performing other tasks in Python , a language that handles a problem like this quite elegantly.

where the term in parenthesis takes into account the fact that only a fraction $(1 - e^{-\tau})$ of the photons from the source are actually scattered[3]. The packet can be "observed" by collecting it and binning its energy into a spectrum array. If we repeat this procedure with the N_p packets, we will gradually build up the observed spectrum. Because of the random nature of the algorithm, this spectrum will possess noise, and we will need a fairly large value for N_p to achieve a reasonable signal to noise[4].

[3] Of course, we could treat this by random sampling as well, and only scatter a packet if $R < 1 - e^{-\tau}$, with R some random number. But that would be computationally inefficient, since $\tau \ll 1$ and you would be wasting a lot of cycles on photon packets that don't scatter.

[4] I used 10^5 in testing, and 10^6 for my final output.

a): Consider the case where we only have electrons of a single energy, $\gamma = 10$. Run the Monte Carlo procedure and plot the spectrum of scattered photons on a log-log plot. Your x-axis can be either eV or Hz, and y-axis should be a monochromatic luminosity, i.e., units of ergs s^{-1} eV^{-1} or ergs s^{-1} Hz^{-1}. The result should resemble the result I drew (but did not derive) in class for single γ scattering. In particular, check that where the spectrum cuts off at high energy makes sense.

b): Now consider the more realistic case where the electrons have a power law distribution in energy, between the bounds $\gamma_{\rm min} = 10$ and $\gamma_{\rm max} = 100$. The probability, in any given scattering, that the electron has an energy γ is

$$P = A\gamma^{-p} \text{ for } \gamma_{\rm min} < \gamma < \gamma_{\rm max} \text{ (zero otherwise)} \tag{4}$$

where A is a constant, and we'll take $p = 2.5$. For each scattering, we should then choose the γ of the electron by randomly sampling from this probability distribution (which can be easily done). Run the Monte Carlo for this case and plot the spectrum due to inverse Compton scattering. Argue that it meets your expectations from class. In particular, overplot the slope of the power law that you expect from analytic arguments.

Comment: Your Monte Carlo code could probably already be used in a simple research paper. For example, you could model the x-ray emission from SN 2011fe and constrain the density of the circumstellar environment, as considered in Horesh et al. 2012 . To improve the code, you would want to relax the assumption that all photons start with the same energy $\epsilon_{\rm in}$. Instead, you can imagine randomly sampling the initial frequency of the photons from a real distribution (e.g., a blackbody). In addition, you could fairly easily generalize your code to treat multiple scatterings, or for the more realistic case where the scattering is not isotropic or inelastic. If you also kept track of the position of the photons, you could even consider the case where the scattering cloud is not spherically symmetric.

Starlight Reflections on a Giant Planet

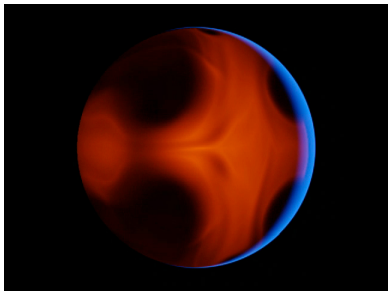

Figure 1: A simulated image of the exoplanet HD80606b, based on hydrodynamical simulations post-processed with radiative transfer calculations. For the paper Laughlin et al. (2009) .

EXTRA SOLAR PLANETS ARE TOO FAR AWAY to image in detail, so we can let our theoretical imagination run wild. The simulated picture of a giant planet in Figure 2 shows a bright blue crescent of reflected starlight, atop the cherry red thermal glow of the heated planet.

The reflected starlight is clearly beautiful, but the actual color and visibility of the crescent will depend on the ratio of scattering to absorptive opacity in the exoplanet atmosphere. Just how shiny is a giant planet? Sudarsky et al.,2000 have performed detailed opacity and radiation transport calculations to determine the albedo (i.e., reflectivity) of jupiter like planets. In this project, we try to understand Sudarksy's results for one particular class of giant planets – those within around 0.05 AU of their star which have equilibrium temperatures of $\sim$ 1500 K. He calls them class IV roasters (great name).

The atmospheres of giant planets are presumed to have essentially solar abundances, with the hydrogen primarily in molecular form. For class IV roasters, the primary scattering opacity in the optical is Rayleigh scattering from hydrogen molecules. The primary absorptive opacity in the optical is from lines – in particular, the resonance lines[5] from alkali metals (i.e., sodium, potassium). Given the relatively high densities of exoplanet atmospheres, the width of these lines is determined not by Doppler broadening, but by pressure (aka collisional) broadening. This pressure broadening is caused by other particles perturbing the atoms, causing shifts in the frequency of bound-bound transitions. Although the full theory can become incredibly involved[6], a reasonable approximation for the shape of a pressure broadened line profile is just the familiar Lorentzian

$$\phi_\nu(\nu) = \frac{\Gamma/4\pi^2}{(\nu-\nu_0)^2 + (\Gamma/4\pi)^2} \tag{5}$$

The width in this case, however, is given by

$$\Gamma = \Gamma_n + \Gamma_p \tag{6}$$

where Γ_n is the natural line width and Γ_p is the pressure broadened line width. Roughly, $\Gamma_p \sim 1/t_{col}$ where t_{col} is the time between collisional perturbations of an atom. Burrows et al., 2000 estimate the pressure widths of alkali metal lines in these giant planets to be approximately $\Gamma_p \sim 10^{11}$ Hz.

a) The strongest resonance lines in this context are the two NaI lines at 5890 Å and 5896 Å (the famous sodium D lines!) and the two KI lines at 7665, 7699 Å. Look up the oscillator strengths of these four

[5] The term *resonance* line just signifies that the lower level of a line is the ground state. Burrows et al., 2000 worked out the theory of pressure broadened alkali metal lines and their relevance in exoplanet atmospheres.

[6] See e.g., 3.3.2 of the Stellar atmospheres book by Rutten.

lines and estimate the cross-section at line center of each, assuming all sodium/potassium is neutral and in the ground state. Given that Na/K are only trace constituents of the atmosphere, it is best to give the cross-section per gas atom (e.g., the cross-section of a sodium atom times the solar number abundance of sodium).

The *single scattering albedo*, a_{ss}, is defined as the probability that a photon scatters in any given interaction with matter[7]

$$a_{ss} = \frac{\sigma_{scat}}{\sigma_{scat} + \sigma_{abs}} \tag{7}$$

A medium with $a_{ss} = 0$ is purely absorbing, and one with $a_{ss} = 1$ is purely reflecting.

[7] Note that Sudarsky and others denote the single scattering albedo by σ, but I didn't want there to be any confusion with our notation for the cross-section. The albedo is a dimensionless quantity.

b) Using the frequency dependent cross-section of Raleigh scattering and of the NaI D line[8] at 5890 Å, calculate and plot the single scattering albedo of a class IV roaster as a function of wavelength in the optical ($\sim 4000 - 8000$ Å). You can compare to the "isolated" model in Figure 7b of Sudarsky et al., to see if your simple model is in the right ballpark.

[8] If you want a better model, of course, you can include *all* of the resonance lines of both sodium and potassium. But to get an idea, one line will do.

Comment: In his figure 7, Sudarsky actually plots up the *spherical albedo*, A_s, of a class IV roaster, defined as the fraction of incident starlight that is reflected at all angles. Because incident photons can scatter multiple times in the atmosphere (and hence have multiple chances of being absorbed) the spherical albedo is generally less than the single scattering albedo. A full radiative transfer calculation is needed to determine the spherical albedo from the single-scattering albedo, which Sudarsky has done in his Figure 2.

Comment: To perform a real calculation of the single scattering albedo, we would have to determine what fraction of the alkali metals are indeed neutral and atomic. For hotter planets, the alkali metals may be partially ionized. For colder planets, the alkali metals may form molecules or condensates. If you look at other plots in Sudarsky et al., you can see that the formation of cloud decks (which are more reflective at all wavelengths) can have a big effect on the albedo of classes of giant planets. The theory of how atoms condense into clouds, however, is still highly uncertain.

Comment: Our calculations suggest that class IV roasters are fairly dark (more like coal than stainless steel). This is somewhat different than Jupiter itself, which has an albedo around 0.5. The albedo of some real giant planets have been constrained; for example the observations of Charbonneau et al., (1999) constrain the (geometric) albedo of one exoplanet to be < 0.3, fairly consistent with our findings.